

---

---

GENERAL  
BIOLOGY

---

## Fluctuating Asymmetry: Trait Variation and the Left–Right Correlation

A. B. Trubyanov and N. V. Glotov

Presented by Academician V.N. Bol'shakov September 18, 2009

Received September 18, 2009

DOI: 10.1134/S0012496610020092

Development of integrated indicators of the environment quality is one of the approaches to environmental monitoring. Fluctuating asymmetry (FA), which is an index of small random deviations from perfect (mirror) symmetry of bilaterally symmetric objects, is such an indicator. Although the phenomenon of FA was discussed in detail back in the early 20th century [1], there is still no unified method of its quantitative estimation [2–6].

All indices used for estimating FA may be divided into two groups: indices based on the absolute difference between the values of trait on the left and right sides (as in various standardizations) and indices based on the product of the values on the left and right sides, which are analogous to the covariance of these values. The index  $Z$  used by Zakharov et al., which coincides with Palmer and Strobeck's index  $F_4$ , to an accuracy of a coefficient, is a widely used index of the first group. The index  $G$  suggested by Gelashvili et al. (it was borrowed from crystallography, where it is used to estimate the pseudosymmetry of crystals) is an example of the second type of indices. They are calculated as follows:

$$Z = \frac{1}{n} \sum_{i=1}^n \frac{|R_i - L_i|}{R_i + L_i},$$

$$G = \frac{1}{n} \sum_{i=1}^n \frac{2R_i L_i}{R_i^2 + L_i^2},$$

where  $R_i$  and  $L_i$  are the values of the trait of the  $i$ th object on the right and left sides, respectively.

We analyzed FA estimated by only one trait in a group of objects. This case is interesting from both the practical point of view [6] and the methodological one, because it has implications for revealing the structure of the studied indices.

Earlier, we studied the characteristics of these two FA indices using a bivariate normal distribution model [7]. We used a computer program to generate pseudo-random numbers that had a bivariate normal distribution with five parameters (the mathematical expectations and variances of trait on the left and right sides and the correlation coefficient between them). The model parameters were based on the measurements of the leaf plate of *Betula pendula* collected by researchers of the Department of Ecology of Nizhni Novgorod State University in the city of Nizhni Novgorod. A successful verification of this model gives us grounds to use it for estimating the FA of the leaf plate of *B. pendula*. Assuming the absence of directed or other types of asymmetry, we assume that the mean values and variances of the trait are the same on the left and right sides; this leaves only three out of five model parameters: the mathematical expectation, variance, and correlation coefficient. However, even this information is redundant: neither  $Z$  nor  $G$  changes significantly if the expectation and variance are varied provided that the variation coefficient is fixed. Thus, it suffices to use two parameters, the coefficient of variation of the trait on the left (or right) side ( $CV$ ) and the coefficient of correlation between the trait values on the left and right sides ( $\rho$ ). In this case, the model adequately describes the FA of specific biological objects.

We used this model to study the dependence of the  $Z$  and  $G$  indices on the aforementioned two parameters. We obtained the following regression equations (in the standardized form):

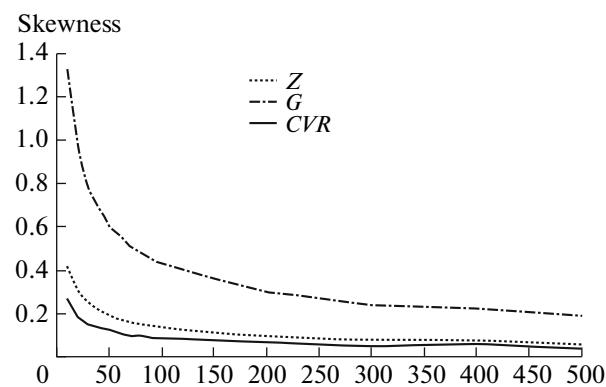
$$Z = 0.93CV - 0.39\rho,$$

$$\text{and } G = 1.43CV - 0.60\rho.$$

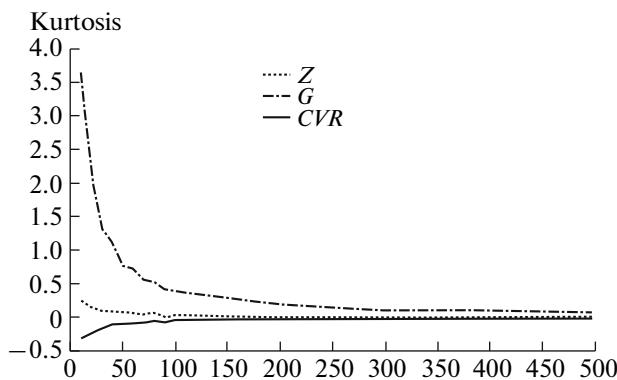
High determination coefficients (0.92 and 0.90, respectively) demonstrate that the model is adequate. As seen from the above equations, the contribution of the trait variation to the FA is more than two times greater than the contribution of the correlation between the trait values on the left and right sides for both indices. Thus, different algorithms of quantitative estimation of FA are based on two characteristics, one of which (the variation coefficient) reflects the

---

Mari State University, pl. Lenina 1, Ioshkar-Ola,  
Republic of Mari El, Russia



**Fig. 1.** Dependence of the skewnesses of the distributions of FA indices on the sample size.

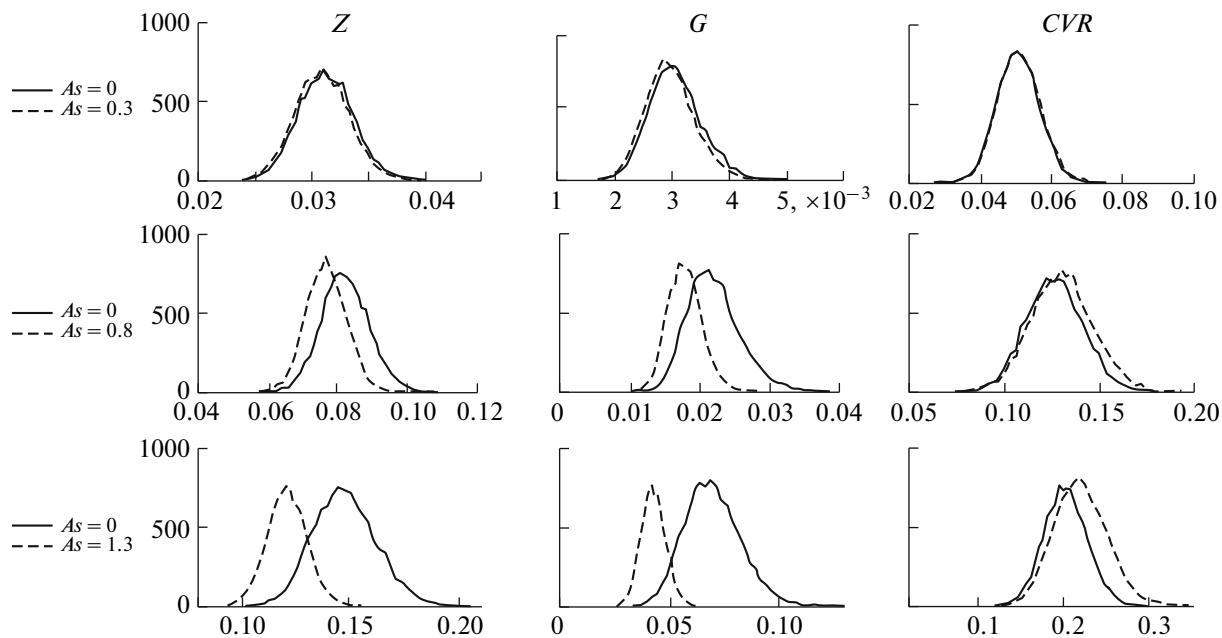


**Fig. 2.** Dependence of the kurtoses of the distributions of FA indices on the sample size.

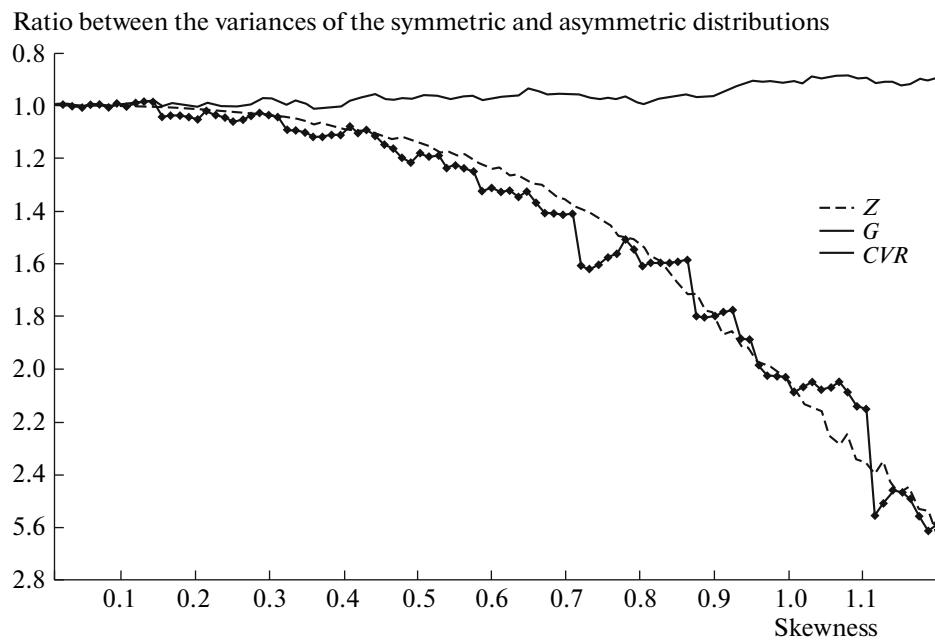
variability of the trait determined by internal or external (with respect to the individual) factors, and the second (the correlation coefficient) is a measure of the structural interrelation and integrity of the individual. These results allow us to put forward a new FA index, namely the “covariance–correlation” index ( $CVR$ ), which explicitly and equally includes both parameters:

$$CVR = CV(1 - \rho^2).$$

How close are the distributions of the statistics  $Z$ ,  $G$ , and  $CVR$  to the normal distribution? We used the Monte Carlo method to study this problem for samples of different sizes in terms of the model of bivariate normal distribution of FA described above. Here, we present, as an example, the result for the following model parameters: mathematical expectation, 25; standard deviation, 5; correlation coefficient, 0.7. We performed 100 000 iterations. The fit of the sample distributions of  $Z$ ,  $G$ , and  $CVR$  to the normal distribution was estimated by the skewness and kurtosis, which are both zero in the case of a normal distribution. As can be seen in Fig. 1, the skewnesses of all the three statistics tends to zero as the sample size increases. However, the skewness of the distribution of  $G$  is much higher: while the skewnesses of  $Z$  and  $CVR$  are about 0.1 at a sample size of 100, the skewness of  $G$  is about 0.2 at a sample size of 500. The kurtoses (Fig. 2) of the distributions of  $Z$  and  $CVR$  tend to zero even more rapidly; the kurtosis of the distribution of  $G$  is especially high for sample sizes smaller than 100. Note that the variation coefficients of the sample distributions of  $Z$ ,  $G$ , and  $CVR$  are relatively low, namely about 5–10% at sample sizes of 100 and larger.



**Fig. 3.** Distributions of the indices  $Z$ ,  $G$ , and  $CVR$  at different skewnesses of the trait distribution.



**Fig. 4.** Changes in the sample variances of the FA indices at different skewnesses of the trait distribution.

When dealing with real biological objects, the distribution of the *original trait* is often asymmetric. Therefore, we have estimated how the skewness of the distribution of the original trait affects the distribution of the FA indices. The Monte Carlo method was used for this estimation.

With increasing skewness ( $As$ ) of the trait, the distributions of all the three parameters shift relative to those for a symmetric distribution; however, this shift is considerably smaller for the *CVR* (Fig. 3). In addition, the variances of *Z* and *G* steadily decrease (Fig. 4), and a decreased variance is known to increase type II error in testing hypotheses on the comparison of the mean values. In contrast, the variance of *CVR* remains almost unchanged.

These results allow us to conclude that our index of FA is the simplest in terms of both the structure and the meaning; its normal distribution gives grounds for calculating not only point but also interval estimates of the FA distribution statistics. In general, this parameter has the best statistical characteristics.

#### ACKNOWLEDGMENTS

This study was supported by the Russian Foundation for Basic Research, project no. 09-04-00 780-a.

#### REFERENCES

1. Astaurov, B.L., *Zh. Eksp. Biol. Ser. A*, 1927, vol. 3, issue 1/2, pp. 1–61.
2. Gelashvili, D.B., Loginov, V.V., Mokrov, I.V., and Silkin, A.A., in *Mater. VII Vseros. Populyatsionnogo seminara* (Proc. VII All-Russian Population Seminar), Syktyvkar, 2004, part 2, pp. 52–62.
3. Zakharov, V.M., *Asimmetriya zhivotnykh* (Animal Asymmetry), Moscow: Nauka, 1987.
4. Zakharov, V.M., Baranov, A.S., Borisov, V.I., et al., *Zdorov'e sredy: metodika otsenki* (Environment Health: Methods of Estimation), Moscow: Tsentr Ekol. Politiki Rossii, 2000.
5. Palmer, A.R. and Strobeck, C., *Annu. Rev. Ecol. Syst.*, 1986, vol. 17, pp. 391–421.
6. Palmer, A.R. and Strobeck, C., in *Developmental Instability (DI): Causes and Consequences*, Oxford: Oxford Univ. Press, 2003, pp. 279–319.
7. Trubyanov, A.B., in *Ekologiya: ot Arktiki do Antarktiki* (Ecology: From the Arctic to the Antarctic) (Proc. Conf. of Young Researchers), Yekaterinburg: Akademkniga, 2007, pp. 321–328.