Methods of Dendrochronology

Applications in the Environmental Sciences

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3.5. Correcting for Trend in Variance Due to Changing Sample Size

S. Shiyatov, V. Mazepa, and E. Cook

With the possible exception of tree-ring chronologies developed from even-aged stands or narrow-age classes of trees, the yearly sample size of m indices in a tree-ring chronology can be expected to diminish backward in time as younger trees drop out of the series. As the sample size decreases below some threshold, commonly between 5 and 10, a perceptible increase in the variance of the mean-value function can be discerned when compared with better replicated time intervals. This increase in variance is largely a function of decreasing sample size, which is independent of changes in the variance owing to environmental influences on radial growth. This problem of nonuniform variance owing to changing sample size was recognized by Schulman (1956). He suggested deleting the early, poorly replicated portions of chronologies because of this non-climatic, statistical artifact. To date, this method of deletion is commonly used in dendroclimatic studies to avoid spurious conclusions concerning past climatic variability.

The change in time series variance, described above, is related to the change in variance of the arithmetic mean, equation (3.23), which is proportional to $1/m$. As m gets small (say, $m < 10$), additional reductions in m result in rapid increases in $1/m$. The change in the standard error of the mean, square root of equation (3.23), is even more dramatic because it is proportional to $1/\sqrt{m}$. How quickly this effect will manifest itself in the mean-value function will depend upon the variance of the sample of m indices, equation (3.22), and the signal-to-noise ratio (SNR) of the chronology, equation (3.15). The SNR and a related measure called the subsample signal strength (SSS) (Wigley *et al.*, 1984) will be described in more detail in Section 3.6. As will be shown, SSS can be used to determine the point in time where a chronology loses too much accuracy to be useful, owing to reduced sample size. For now, it is sufficient to illustrate the sample-size effect on the variance of tree-ring chronologies and describe a method developed by Shiyatov and Mazepa (1987) for correcting this effect.

Figure 3.11 (Shiyatov and Mazepa, 1987) shows the way in which the coefficient of variation of a tree-ring chronology can vary as a function of sample size. These plots were created, from an ensemble of 22 indexed *Picea obovata* tree-ring series, by randomly selecting many subsets of size $m = 1 - 20$ from the total and by computing the coefficient of variation of each mean series. Below about $m = 7$, there is a clear increase in the plots of the mean and maximum coefficients of variation. This reflects the increase in variance owing to decreasing sample size. The plot of minima remains much more constant in the range of $m = 2 - 10$. This probably reflects a subset of series in the ensemble that are highly correlated and, therefore, have high SNR. Nonetheless, *Figure 3.11* indicates that a trend in variance from decreasing sample size should be expected most of the time.

Shiyatov and Mazepa (1987) note that there are times when it is important to use as much of a tree-ring chronology as possible. Given this circumstance, they suggest the following method for correcting the trend in variance caused by

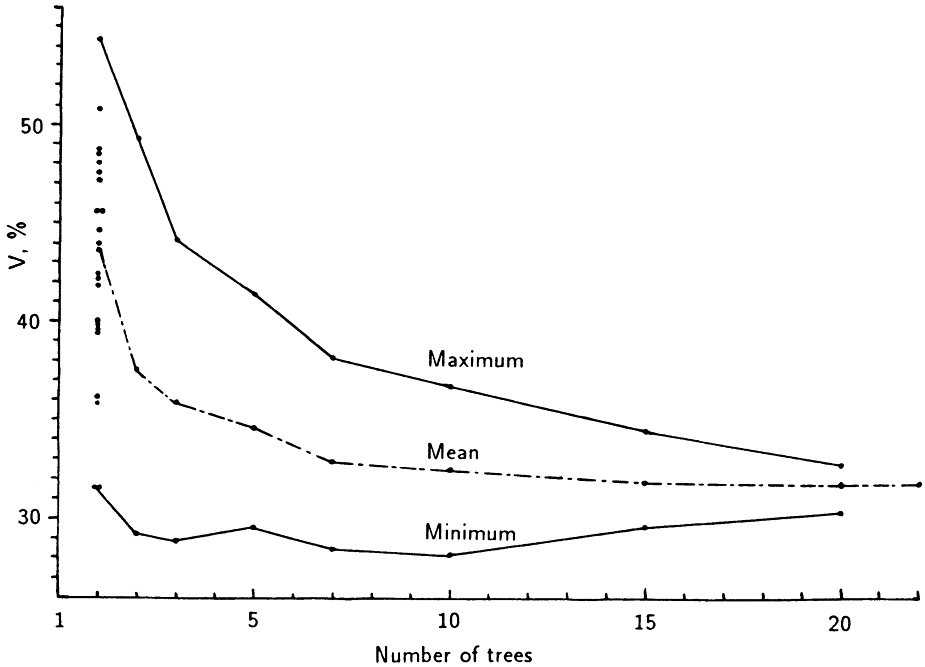

Figure 3.11. The maximum, mean, and minimum values of the coefficient of variation (CV) obtained from a different number of sampled *Picea obovata* trees from the same site. The dots (·) on the left of the chart indicate the range of the CV of the individual series.

changing sample size. For the time interval of maximum sample size, m_{max}, compute the coefficient of variation for the mean series as

$$CV_{std} = s_{std}/\bar{x}_{std} \quad , \tag{3.35}$$

where s_{std} and $\bar{x}_{std}$ are the standard deviation and mean, respectively, of that series. CV_{std} is the standard used for contracting the variance in other time periods having smaller sample sizes. Having estimated CV_{std}, new coefficients of variation CV_k are then computed using equation (3.35) for the total time period of the longest series, the entire period of overlap of the mean of the two oldest series, the entire period of overlap of the mean of the three oldest series, and so on up to the time interval and sample size covered by CV_{std}. The variance corrections for the time intervals not having maximum sample size are then computed as

$$I_t^{cor} = (I_t^{act} - \bar{I}) * k + \bar{I} \quad , \tag{3.36}$$

where I_t^{cor} is the corrected tree-ring index, I_t^{act} is the uncorrected index for year t, I is the the mean of the entire mean series, and k is the coefficient of contraction estimated as $k = CV_{std}/CV_k$, which is the ratio of the coefficient of variation of the m_{max} time period to that for a sample size $m < m_{max}$.

Table 3.2 shows the way in which CV_k varies in an ensemble of tree-ring indices of 24 *Picea obovata*. The last value for all sampled trees is CV_{std}. These values reinforce the example in *Figure 3.11* and, again, indicate that sample-size effects on variance are likely to occur at some point below $m = 10$. Having estimated the necessary CV_{std} and CV_k in *Table 3.2*, Shiyatov and Mazepa (1987) used this information in equation (3.36) to produce a corrected tree-ring chronology. The uncorrected and corrected series are shown in *Figure 3.12*. There is a clear contraction in the variance of the corrected series below $m = 5$, in accordance with the values in *Table 3.2*.

Table 3.2. Change in the coefficient of variation (CV) of a mean chronology (*Picea obovata*) owing to changing sample size. The % deviation of each CV from the standard value is used to correct variance of the mean series for changing sample size.

Sample size	*Time interval*	*Years*	*CV in %*	*% Deviation of CV from the standard*
1	1829–1968	140	58.5	+20.3
2	1837–1968	132	57.2	+19.0
3	1854–1968	115	49.9	+11.7
5	1876–1968	93	46.0	+7.8
6	1878–1968	91	43.6	+5.4
7	1879–1968	90	42.4	+4.2
8	1881–1968	88	41.2	+3.0
9	1882–1968	87	39.5	+1.3
10	1884–1968	85	38.4	+0.2
11	1885–1968	84	38.8	+0.6
12	1893–1968	76	37.1	-1.1
All sampled trees – The standard				
13–24	1894–1968	75	38.2	0.0

A trend in variance, which is independent of changing sample size, can also be expected in tree-ring chronologies of ring-porous tree species, such as *Quercus*. The active xylem vessels of ring-porous species are almost totally restricted to the newly formed vessels of each growing season. These large *springwood* vessels are formed from stored carbohydrates before the flush of new leaves each year. Their contribution to each total ring width varies little from year to year, compared with the subsequently formed *summerwood* vessels, and declines more slowly with age than the summerwood width. Thus, the contribution of springwood vessels to the total ring width often increases with age. This causes the yearly variance of ring-porous radial increments to decrease with age in a way that is independent of sample size.

As a means of correcting this problem in *Quercus* tree-ring chronologies, Cook (unpublished) included in the tree-ring standardization program ARSTND

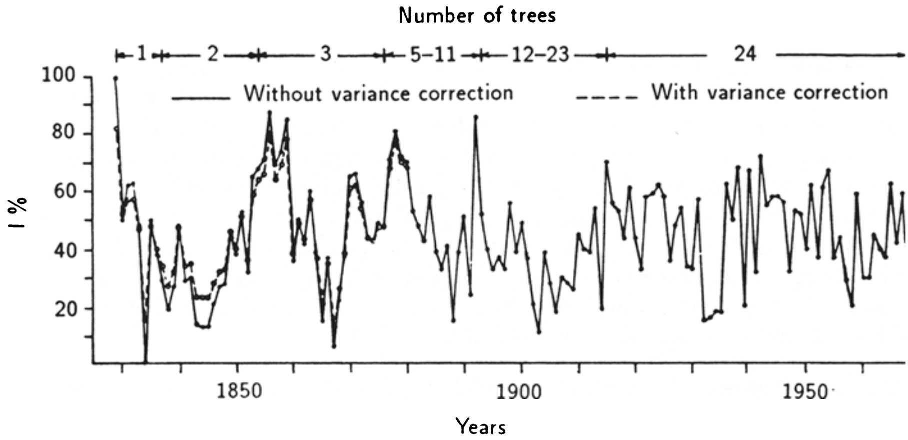

Figure 3.12. The mean chronology of 24 ***Picea obovata*** tree-ring series without (solid line) and with (dashed line) the variance correction based on the coefficient of variation. The change in sample size in the chronology is indicated by the number of trees at the top of the plot. The most obvious contraction of variance occurs below a sample size of five trees.

(Cook and Holmes, n.d.) an option for removing the trend in variance in tree-ring chronologies. This method is distinctly different from that of Shiyatov and Mazepa (1987) and does not rely on the coefficient of variation. It is based on fitting a smoothing spline to the absolute values of the mean-corrected, standardized tree-ring indices. The spline is used to track the trend in the standard deviation of the indices, as revealed by the low-frequency fluctuations in the absolute values. Each absolute value is divided by its respective spline value. The corrected absolute values are than back-transformed into normal tree-ring indices and scaled to have the same overall variance as the original, uncorrected indices. This technique can be used on both single series and the mean-value function. In the latter case, it will correct for both the trend in variance caused by ring-porous wood anatomy and the changing sample size, if the variance correction is not made on the individual series.

Given that the method of Shiyatov and Mazepa (1987) or the method in the *ARSTND* program (Cook and Holmes, n.d.) can be used to correct for a trend in variance owing to changing sample size, a word of caution must be made. The increase in variance when m gets small is a consequence of the noise in the ensemble that is not adequately reduced by averaging. As a result, those mean values based on small m are considerably less precise and probably less accurate then the mean indices derived from larger sample sizes. Correcting the variance for changing sample size will not necessarily improve the accuracy. It merely adjusts those means based on $m < m_{max}$ to behave as if they were based on the maximum sample size. As a consequence, the loss of accuracy is masked by the variance correction. Additional information on the accuracy of the small *m* segment of tree-ring chronologies can be obtained by computing the SSS (subsample signal strength) and should be made available if corrections are made to the variance for changing sample size.

3.6. Basic Chronology Statistics and Assessment

K. Briffa and P.D. Jones

3.6.1. Measuring the statistical quality of a chronology

This section is concerned with the problem of assessing the statistical confidence of a chronology. Up to this point we have described various approaches and techniques that are used to produce tree-ring chronologies from such variables as ring widths or maximum latewood density data. We have discussed in some detail the ways in which measured series of these data may be standardized during chronology production so that the unwanted index time series, thought to obscure those variations representing the hypothesized forcing with which we are concerned, has been removed. In other words, we have described the production of chronologies in terms of a noise-reduction process, and the concepts of signal and noise have been clearly framed in terms of specific hypotheses or applications.

Here, we will investigate the concept of a chronology signal. This signal is a statistical quantity representing the common variability present in all of the tree-ring series at a particular site. The strength of this signal is estimated empirically, and here it is not necessary to speculate on the forcing(s) of which the signal is the manifestation. We merely assume that a group of tree-ring series from a site make up one sample from a hypothetical population whose average would be the *perfect* chronology – one that fully represents the underlying forcing(s). The variance of any series of tree-ring indices will contain this common forcing signal (though it will be modified according to how the data are standardized), but in any one core it will be obscured by variability common only to the specific tree and core – the statistical noise. By definition, this noise is uncorrelated from core to core and, therefore, will cancel out in a chronology to an extent that depends on the number of series being averaged. Therefore, the question of the statistical quality of a chronology may be phrased as follows: To what degree does the chronology represent the hypothetical population chronology? To answer this question, it is necessary, first, to estimate the strength of the signal and, second, to quantify how clearly this signal is expressed in the chronology. We shall discuss these points in turn.

3.6.2. Estimating the chronology signal strength

The Analysis of Variance Technique

Traditionally, dendroclimatologists haved used the Analysis of Variance (ANOVA) technique to estimate signal and noise within groups of standardized