Methods of Dendrochronology

Applications in the Environmental Sciences

Edited by

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This definition of G_t suggests that the common climatic component, C_t, may be the signal of interest, since A_t, $D1_t$, and $D2_t$ are considered collectively as non-climatic variance or noise. These definitions of signal and noise are implicit when standardizing tree-ring series for dendroclimatic studies (Fritts, 1976). They will be maintained throughout this section on tree-ring standardization and chronology development, with the realization that other applications of tree-ring analysis may define signal and noise differently.

3.3. Tree-Ring Standardization and Growth-Trend Estimation

E. Cook, K. Briffa, S. Shiyatov, and V. Mazepa

3.3.1. Introduction

The estimation and removal of G_t from a ring-width series has been a traditional procedure in dendrochronology since its modern-day development by A.E. Douglass (1914, 1919). This procedure is known as standardization (Douglass, 1919; Fritts, 1976). Early workers searching for climatic signals in the ring-width series of old conifers identified long-term growth trends in measured ring-width data that could confidently be attributed solely to tree aging. In general, many factors can influence tree growth. However, by careful selection of long-lived trees growing in climatically stressed sites, early workers attempted to ensure that non-climatic growth influences such as competition and defoliation were minimized (Douglass, 1914). In this way, the selected trees were likely to have a strong climate signal, and the ring-width series were such that unwanted noise (resulting from tree aging) could be unambiguously identified. Schulman (1945b) described the purpose of standardization as follows:

> To obtain a mean curve representing trees of various ages, the usual procedure is to "standardize" the individual tree curves by computing percentage departures from a trend line fitted to the curve and then to average the standardized values. Thus the large average growth rate of youth is reduced to conform with slower growth of maturity and old age.

From this quote, it is clear that the original intent of standardization was two-fold: (1) to remove non-climatic age trends from the ring-width series and (2) to allow the resultant standardized values of individual trees to be averaged together into a mean-value function by adjusting the series for differential growth rates due to differing tree ages and differences in the overall rate of growth. The "percentage departures" described by Schulman (1945b) are commonly known today as "tree-ring indices" (Fritts, 1976). Fritts and Swetnam (1986) provide an excellent description of the sequential process of trend removal, indexing, and averaging used in dendrochronology.

Standardization transforms the nonstationary ring widths into a new series of stationary, relative tree-ring indices that have a defined mean of 1.0 and a relatively constant variance. This is accomplished by dividing each measured ring width by its expected value, as estimated by G_t. That is,

$$I_t = R_t / G_t \tag{3.1}$$

where I_t is the relative tree-ring index. When standardizing ring-width data, the indices are produced by division instead of differencing because ring-width series are heteroscedastic. That is, the local variance of ring widths is generally proportional to the local mean, where local is defined as some subinterval of time within the time span covered by the ring widths. The actual relationship is usually positive and linear between the ring-width means and their standard deviations when compared over time. *Figure 3.4(a)* shows a plot of 10-year mean ring widths versus their standard deviations for a collection of lodgepole pine (*Pinus contorta*) ring-width series. The positive correlation (r = .67) is quite apparent. *Figure 3.4(b)* shows the same data after the ring-width series have been standardized using negative exponential and linear regression curves. The correlation between mean and standard deviation (r = .10) is now largely gone.

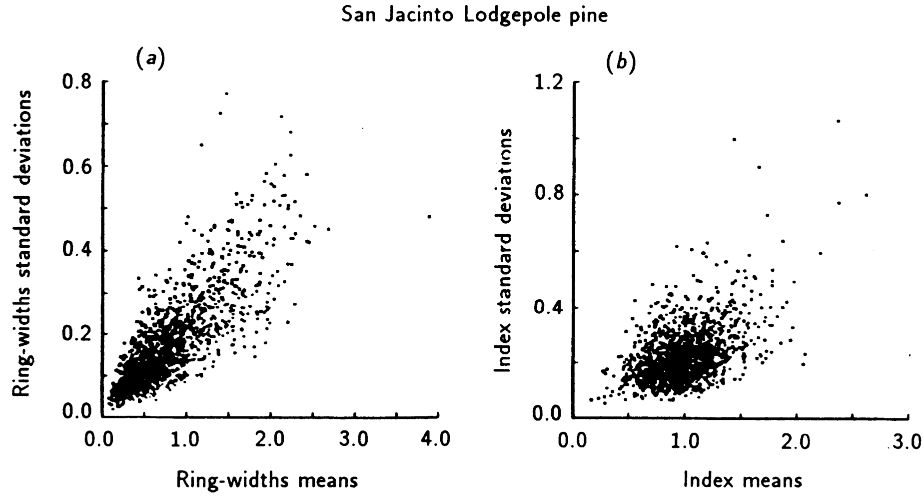

Figure 3.4. The relationship between the mean and standard deviation of tree-ring data before and after standardization. *Figure 3.4(a)* shows the scatter-plot of 10-year mean ring widths versus their standard deviations. The correlation is r = .67. In *Figure 3.4(b)*, after standardization and reduction of the ring widths to indices, the linear dependence between the mean and standard deviation is for the most part gone (r = .10).

Another way of stabilizing the variance is by transforming the ring widths to logarithms. In this case, the resultant indices are computed by subtracting $\log_e G_t$ from $\log_e R_t$, not by dividing as above. That is,

$$\log_e I_t = \log_e R_t - \log_e G_t \quad . \tag{3.2}$$

Problems arise in employing the logarithmic transformation to ring widths when the series has locally absent rings, which are typically coded as zero when using equation (3.1) to estimate I_t. Since the logarithm of zero does not exist, it has been suggested (e.g., Warren, 1980) that some arbitrarily small positive number replace the zero. This procedure may impart a statistically significant negative skew to the probability distribution of $\log_e I_t$ if the chosen number is too small. In addition, a logarithmic transformation of the form in equation (3.2) may overcorrect the heteroscedasticity and impart a negative dependence between the mean and standard deviation. Preliminary research indicates that the correct transformation of ring-width series to stabilize the variance is $\log_e(R_t + c)$, where c is a constant estimated from the series being transformed. That a simple logarithmic transformation will not be correct all of the time is illustrated in *Figure 3.5* for two white oak (*Quercus alba*) ring-width series. The lower series shows the typical linear dependence between the local 10-year means and their standard deviations [$r = .77$, *Figure 3.5*(*b*)]. A logarithmic transformation is clearly justified. In contrast, the upper ring-width series from the same stand of trees has no significant linear dependence between the local means and standard deviations [$r = .08$, *Figure 3.5*(*a*)]. Therefore, a logarithmic transformation is not justified in this case. These examples indicate the need for additional research to determine the best way to stabilize the variance of tree-ring indices by transformation.

The method of computing tree-ring indices as ratios has a long history in dendrochronology (Douglass, 1936; Schulman, 1956; Fritts, 1976). However, with the development of X-ray densitometry, additional tree-ring variables can be measured that also require some form of standardization. Bräker (1981) analyzed the properties of six tree-ring variables (total ring width, earlywood width, latewood width, latewood percentage, minimum earlywood density, and maximum latewood density) for conifer tree species growing in Switzerland. He found that latewood percentages and density data were properly standardized as differences from the estimated growth trend, not as ratios. Bräker (1981) justified the computation of differences because latewood percentages and density data do not usually show a clear dependence between mean and variance. However, Cleaveland (1983) found that maximum latewood densities of semiarid-site conifers in the western United States still required indexing by the ratio method to stabilize the variance. As more experience is gained in standardizing these comparatively new tree-ring variables, a preferred method of computing indices may be determined.

Because tree-ring indices are stationary processes having a defined mean and homogeneous variance, the index series of many trees from a site can be averaged together to form a mean-value function. Strictly speaking, it is not valid to average together nonstationary processes, like the majority of ring-width series, because such processes do not possess a defined mean or variance. While it is possible to compute the mean and variance of a ring-width series for a specific time period, it is incorrect to use these statistics as expectations for other

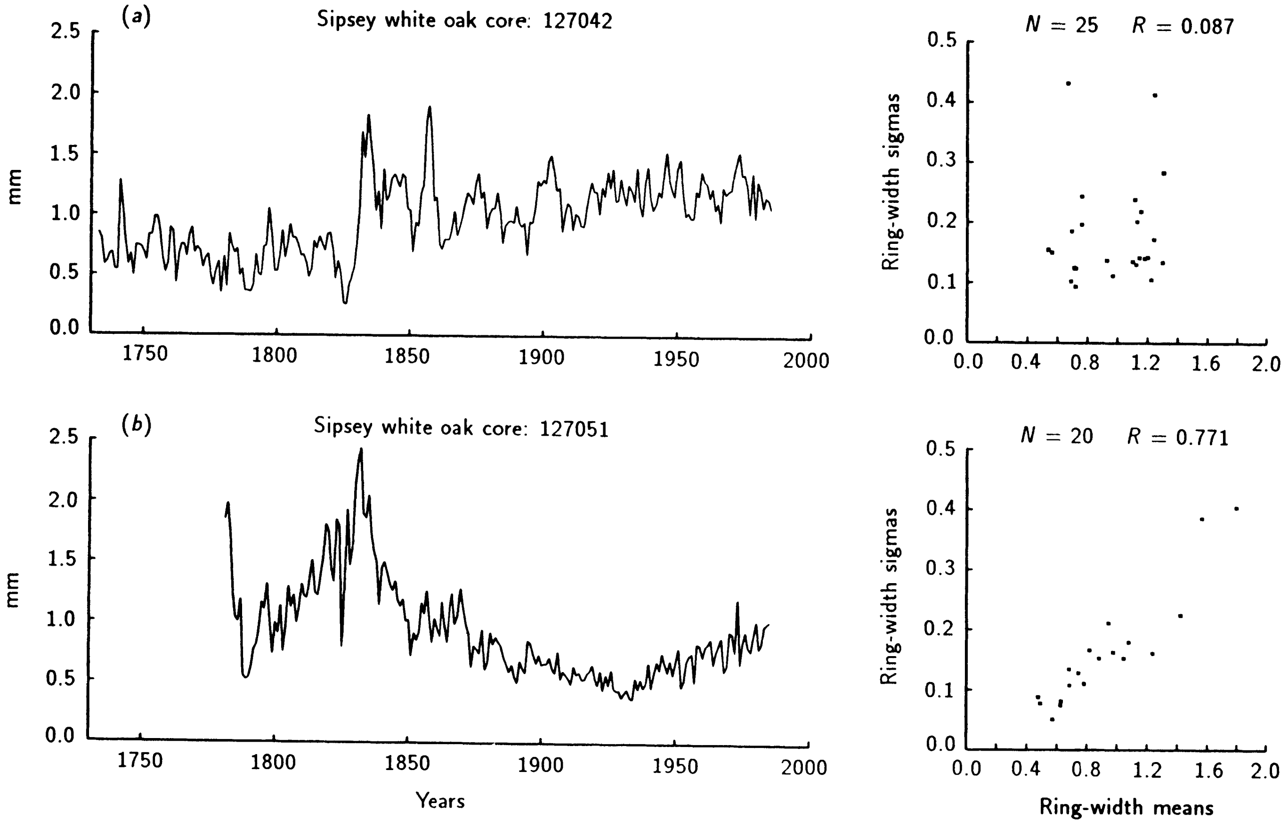

Figure 3.5. The variability in the mean-to-standard deviation relationship in two white oak (*Quercus alba*) ring-width series from the same stand. The bottom series (*b*) has the normally expected linear dependence, while the upper series (*a*) has no significant relationship. A logarithmic transformation to stabilize the variance would be appropriate for the lower series, but not for the upper series.

time periods or to use them as within-series estimates of the population parameters. This same problem carries through in a cross-sectional or between-series sense, where the lack of a defined sample mean makes the concept of estimating the between-series population mean invalid.

The mean-value function of tree-ring indices is frequently used to study past climate (Fritts, 1976). When climate (i.e., C_t in the linear aggregate model) is the signal of interest, all other information in the ring widths (i.e., A_t, $D1_t$, $D2_t$, and E_t) is considered as noise and discarded as a compound form of G_t or minimized through averaging into the mean-value function.

Many methods are available for estimating G_t. They fall into two general classes: deterministic and stochastic. The deterministic methods typically involve fitting an *a priori* defined mathematical model of radial growth to the ring-width series, by the method of least squares. Ordinarily, it is assumed that $G_t = f(A_t)$, with $D1_t$ and $D2_t$ absent or negligible. The stochastic methods are more data-adaptive, and often are chosen by *a posteriori* selection criteria. These methods allow for the more general case, $G_t = f(A_t, \delta D1_t, \delta D2_t)$. As will be seen, each class of models has its advantages and disadvantages.

3.3.2. Deterministic methods of growth-trend estimation

The simplest deterministic model is the linear trend model, *viz.*,

$$G_t = b_0 + b_1 t \quad , \tag{3.3}$$

where b_0 is the y-intercept, b_1 is the slope of the fitted linear regression line, and t is time in years from 1 to n. *Figure 3.1(c)* shows a linear trend line fit to a ring-width series. The slope coefficient, b_1, may be constrained to be negative or zero if the *a priori* expectation of G_t requires it. However, as noted earlier, G_t may also be negative exponential in form because of the *geometrical constraint* argument. Therefore, Fritts *et al.* (1969) suggest fitting the modified negative exponential curve of the form

$$G_t = a \exp^{-bt} + k \quad , \tag{3.4}$$

where a, b, and k are coefficients of this nonlinear regression function, all a function of time t. *Figure 3.1(a)* shows an example of this curve fit. Other functions have been used for estimating the age trend of ring-width series, such as the negative exponential curve (Fritts, 1963)

$$G_t = a \exp^{-bt} \quad , \tag{3.5}$$

which is a special case ($k = 0$) of equation (3.4); the hyperbolic function (Eklund, 1954)

$$1/G_t = a + b\,(t - k) \quad , \tag{3.6}$$

where k is the middle year (i.e., $k = n/2$) of the series; the power function (Kuusela and Kilkki, 1963)

$$G_t = at^{-b} \quad , \tag{3.7}$$

the generalized exponential or "Hugershoff" function (Warren, 1980; Bråker, 1981)

$$G_t = at^b \exp^{-gt} \quad ; \tag{3.8}$$

and the Weibull probability density function (Yang *et al.*, 1978)

$$G_t = at^{a-1}b^{-a} \exp[-(t/b)^a] \quad . \tag{3.9}$$

Equations (3.8) and (3.9) are able to fit both the juvenile increase in radial growth and the subsequent exponential decay of ring width as trees mature. These functions are more theoretically complete than the other models, which can only fit the maturation phase of declining radial growth.

These deterministic models produce monotonic or unimodal curves, which clearly require that the observed growth trend be simple in form. However, Warren (1980) made his fitting procedure much more general by allowing the age trend to be modeled as a temporal aggregate of generalized exponential functions. This allowed Warren (1980) to fit a series of suppression-release events in his tree rings. And, except for equations (3.3) and (3.4), all have a limiting value of zero for G_t as t grows large. This property is unsatisfactory for many trees that approach a constant level of ring width in old age, hence the preference for equation (3.4) over equation (3.5) by Fritts *et al.*, (1969) for standardizing semiarid-site, old-age conifers. The deterministic growth-trend models described above are most appropriate for open-canopy stands of undisturbed trees and for young trees with strong juvenile age trends. It is also clear that these models only depend on time t for predictive purposes. Thus, they are deterministic. The last two models [equations (3.8) and (3.9)] have not been widely used in dendrochronology, although their use in forest mensuration to estimate growth increment functions is more common.

Another family of deterministic growth-trend models is found in polynomial detrending (Jonsson and Matern, 1974; Fritts, 1976; Graybill, 1979). This model for G_t has the form

$$G_t = b_0 + b_1 t + b_2 t^2 + \cdots + b_p t^p \quad . \tag{3.10}$$

The linear-trend model is just a special case of this polynomial model. This method of growth-trend estimation is not based on any *a priori* age-trend model. Rather, a best-fit, order-p polynomial, which is initially unknown, is fit to a ring-width series based on the behavior of that series alone. It is far more *ad hoc* and data adaptive than the previous models, although it still maintains its dependence on time alone for predictive purposes. Polynomial detrending of the form above suffers from problems of order selection, potentially severe end-fitting problems, and poor local goodness-of-fit (Cook and Peters, 1981; Briffa, 1984; Cook, 1985). In spite of their wide usage in the past, polynomials of the form of equation (3.10) are not recommended as a general method for detrending ring-width series.

In the rare situations where the age trend is known to have a simple deterministic form based on theoretical considerations, the best approach is to describe that trend with an appropriate mathematical model. This is the most concise and directly communicable way of describing and explaining the character of the age trend. Nevertheless, assuming a mathematical function for describing trends in tree growth may be unnecessarily restrictive. Problems increasingly arise when this approach is used in situations where trends are complex or where it is necessary to remove relatively medium or short time scale variation in tree-ring data during standardization. Functional growth equations are often too simple.

A more basic weakness of the method is that the goodness-of-fit varies with time because of time-dependent stochastic departures from the theoretical model. Thus, noise-related medium-frequency variance may be retained in some parts of the series, yet removed in others. This may introduce spurious medium frequency variations into the standardized series.

Given the inherent limitations and potential problems of deterministic growth-trend models in fitting the low- and middle-frequency stochastic perturbations commonly found in ring-width series (see *Figures 3.1–3.3*), stochastic methods of growth-trend estimation have been investigated. These methods fall within the realms of low-pass digital filtering (Parker, 1971; Cook and Peters, 1981; Briffa *et al.*, 1983), exponential smoothing (Barefoot *et al.*, 1974), and differencing (Box and Jenkins, 1970). These methods will be described next.

3.3.3. Stochastic methods of growth-trend estimation

Low-Pass Digital Filtering

Low-pass digital filtering typically involves passing an odd-numbered set of symmetrical, low-pass filter weights over a ring-width series to produce a smoothed estimate of the actual series. This is accomplished as

$$G_t = \sum_{i=-n}^{+n} w_i R_{t+i} \Big/ \sum_{i=-n}^{+n} w_i \quad , \tag{3.11}$$

where G_t is the tth filtered value and where w_i is the weight by which the value of the series i units removed from t is multiplied. There are $2n+1$ filter weights. Equation (3.11) clearly shows that this digital filter is a centrally weighted moving average of the actual data. The w_i tapers off equally on both sides of the maximum central weight until the outermost weights are arbitrarily close to zero. These weights may be derived from the Gaussian probability distribution function and tailored to have a particular frequency response (Mitchell *et al.*, 1966; Briffa, 1984). Alternately, the cubic-smoothing spline can be used as a symmetrical, low-pass digital filter (Cook and Peters, 1981), without the need to compute explicitly the filter weights.

Symmetrical filters of this type preserve the original phase information of the unfiltered time series in the filtered values. In the past, equally weighted (i.e., $w_i = 1.0$ for all i) moving averages have been advocated for standardizing tree-ring series (e.g., Bitvinskas, 1974). This form of digital filter cannot be recommended under any circumstances because it causes undesirable phase shifts in the smoothed age-trend estimates and distortion in the power spectrum of the resultant standardized tree-ring indices. Fritts (1976) and Briffa (1984) detail the use of digital filters in tree-ring analysis.

The degree of smoothness of the low-pass filter estimates of G_t depends on the characteristic frequency response of the filter. For the Gaussian filter, the response is approximately

$$u(f) = \exp(-2\pi^2 \, s_G f^2) \quad , \tag{3.12}$$

where s_G equals $L/6$ and L is the length of the filter (Briffa, 1984). For the cubic-smoothing spline (Cook and Peters, 1981), the frequency-response function is computed as

$$u(f) = 1 - \frac{1}{1 + \dfrac{p\,(\cos 2\pi f + 2)}{6\,(\cos 2\pi f - 1)^2}} \quad , \tag{3.13}$$

where p is the Lagrange multiplier that uniquely defines the frequency response of the spline. The 50% frequency-response cutoff, which is the frequency at which 50% of the amplitude of a signal is retained (or removed), is typically used to define the degree of smoothing by a digital filter. For the smoothing spline, it can be defined in terms of p as

$$p = \frac{6(\cos 2\pi f - 1)^2}{\cos 2\pi f + 2} \quad . \tag{3.14}$$

Equations (3.12)–(3.14) assume a sampling interval of one year, which is typical for tree-ring series. *Figure 3.6* shows some characteristic frequency-response functions of the Gaussian filter and smoothing spline. The transition bandwidth of each filter, which is defined as the bandwidth of frequencies between $u(f) = 0$ and $u(f) = 1$, is rather broad in each case. The response of the spline filter is steeper than that of the Gaussian filter, which means that the latter will leave in slightly more low-frequency variance for a given 50% response cutoff. Although the transition bandwidths in *Figure 3.6* could be criticized for being too broad, they are probably satisfactory for estimating G_t – given the considerable uncertainty in knowing what degree of smoothing to use.

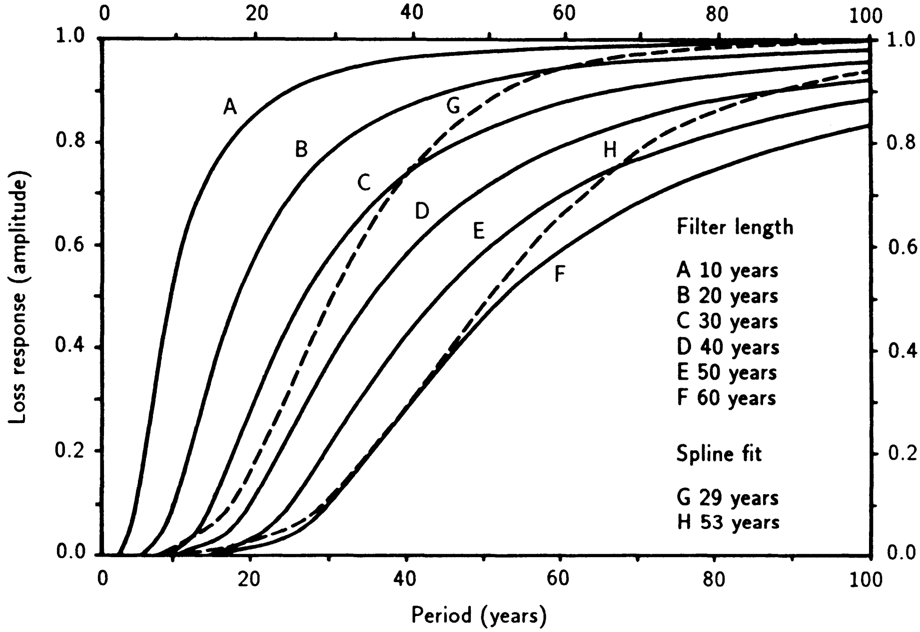

Figure 3.6. Some characteristic frequency-response functions of the Gaussian filter (solid curves) and the cubic-smoothing spline (dashed curves). The 50% frequency-response cutoff in years is given for each filter. (Modified from Briffa, 1984.)

There is often little theoretical basis for selecting the *proper* degree of curve flexibility or data smoothing when using digital filters. For this reason, a chosen digital filter can be rather difficult to justify. Briffa *et al.* (1983) used *a priori* information about stand-management practices in Europe to select the frequency response for their low-pass filter used in estimating G_t. Their selection criterion is based on the concept that the unwanted noise in the ring widths is frequency dependent (Briffa *et al.*, 1986). In this case, it was felt that the noise was largely restricted to wavelengths longer than about 50 years. A similar determination was made by Cook and Peters (1981) and Blasing *et al.* (1983), based on using the cubic-smoothing spline as a digital filter on ring widths from North American trees. However, the filters advocated by these studies may be too flexible for general use.

Given that the frequency dependent properties of the noise (i.e., the band-limited spectral properties of the age trend and stochastic disturbance effects) will be unknown *a priori* in many situations, how might the *appropriate* or *optimal* frequency response be selected objectively? One possible criterion is based on the signal-to-noise ratio (SNR) (Wigley *et al.*, 1984). The SNR is defined as

$$\mathrm{SNR} = N\bar{r}/(1 - \bar{r}) \quad , \tag{3.15}$$

where $\bar{r}$ is the average correlation between trees and N is the number of trees in the ensemble of standardized tree-ring indices. SNR is an expression of the strength of the observed common signal among trees in the ensemble. Tuning the frequency response of the digital filter to maximize the SNR would seem to be one optimal and objective criterion. Two examples, in *Table 3.1*, show how the SNR of tree-ring indices can change as the frequency response of a Gaussian filter is tuned to maximize that criterion. In example A, a peak in SNR is indicated for the filter with a 50% frequency response of 40 years. However, for example B, the SNR increases all the way to the minimum 50% frequency response of 10 years. This result indicates that there may not be a clear maximum SNR in the low-to-intermediate frequencies of some tree-ring chronologies, which will make the application of the maximum SNR criterion more difficult. The maximum SNR criterion is also flawed because it assumes, in the derivation of equation (3.15), that the series being cross-correlated is serially random. This means that the SNR is best suited for measuring the strength of the observed high-frequency signal in the tree-ring indices, not the persistent, low-frequency signal that may of interest in the study of climatic change. Thus, the maximum SNR criterion may be biased toward selecting a digital filter that removes an excessive amount of low-frequency variance during standardization.

Table 3.1. The effect of different Gaussian low-pass filters on the fractional common variance ($\bar{r}$) and signal-to-noise ratio (SNR) of an ensemble of standardized tree-ring indices (Briffa, 1984). The tree-ring data are from oak sites in the United Kingdom. The 50% frequency response of each filter is given in years. Each analysis is based on 14 trees for the time period 1880–1979.

Example A			*Example B*		
Filter length	$\bar{r} \times 100$	*SNR*	*Filter length*	$\bar{r} \times 100$	*SNR*
10 years	30.09	6.03	10 years	46.54	20.03[a]
20 years	30.90	6.26	20 years	41.93	16.61
30 years	32.08	6.61	30 years	39.18	14.81
40 years	32.83	6.84[a]	40 years	38.06	14.14
50 years	32.77	6.83	50 years	37.53	13.82
60 years	32.42	6.72	60 years	36.94	13.47

[a]Denotes the maximum $\bar{r}$ and SNR.

Blasing *et al.* (1983) examined another objective criterion for choosing the optimum filter response of the smoothing spline. They selected a filter for standardization that produced the best tree-ring chronology for dendroclimatic reconstruction. In their example, Blasing *et al.* (1983) reconstructed total annual precipitation for Iowa from white oak (*Quercus alba*) tree-ring chronologies standardized with splines of differing frequency response. They determined the optimum frequency response from the chronology, which produced the reconstruction that verified best against independent data. In this case, the optimum spline had a 50% frequency response of about 50 years, a value similar to example A in *Table 3.1*. It is not clear whether this technique will be widely applicable, however, because it depends on having long, homogeneous climatic records for reconstruction and verification. This will not be the case for many regions of the world where tree-ring analysis can be done. In addition, this optimization rule depends on having tree-ring data that can strongly model the climatic data. This may not be the case in many mesic forest environments where climate has a weaker and less-direct impact on tree growth than was the case for the Iowa oaks.

Another possible criterion for objectively selecting an optimal digital filter for standardization is related to the concept of "trend in mean" (Granger, 1966). From the theory of spectral analysis (Jenkins and Watts, 1968), the lowest-frequency harmonic that can be be theoretically resolved in a time series has a frequency $f = 1/n$, where n is the length of the series. This is the fundamental frequency of the process (Jenkins and Watts, 1968). The fundamental frequency corresponds to one complete sine wave with a cycle length of n years. According to the definition of trend in mean, any variance at wavelengths longer than the observed time series ($f < 1/n$) cannot be differentiated from pure trend ($f = 0.0$), unless strong *a priori* information on climatic variability and the properties of the ring-width data allow for it. For example, if it is known that the sampled trees are temperature sensitive and that the trend in temperature for the recent past has been positive for the region where the trees were sampled, then any positive trend in ring width may be related to the positive trend in temperature. In this case, ring-width series with positive trends should be standardized with deterministic curves that are constrained to be non-positive in trend. Obviously, care must be taken to ensure that the observed positive trend in ring width is not due to non-climatic effects, such as a release from competition. However, for the majority of cases, trend in mean can be used as the basic definition of the theoretical resolvable limit of climatic information in tree-ring chronologies.

Given the above definition of trend in mean, another objective criterion for selecting the optimal frequency response of a digital filter is as follows. Select a 50% frequency-response cutoff in years for the filter that equals some large percentage of the series length, n. This is the $\%n$ criterion described in Cook (1985). The results of Cook (1985) suggest that the percentage is $67\%n$ to $75\%n$ based on using the cubic-smoothing spline as a digital filter. The $\%n$ criterion ensures that little low-frequency variance, which is resolvable in the standardized tree rings, will be lost in estimating and removing the growth trend. This criterion also has a bias of sorts because of the stiff character of the low-pass filter estimates of the growth trend. It will not necessarily guarantee and, in fact, will rarely possess any kind of optimal goodness-of-fit. Thus, the SNR of an ensemble of tree-ring indices standardized by the $\%n$ criterion will usually be inferior to those standardized by the maximum SNR criterion. The SNR of $\%n$ standardized indices can, of course, be increased by adding additional trees to the ensemble. Additional increases in SNR are also possible by prewhitening the detrended indices as autoregressive processes and by using a robust mean in estimating the mean-value function (Cook, 1985). The last two approaches will be described in Sections 3.4.2 and 3.4.3. Either way, the drawback of lower SNR for the $\%n$ method can be ameliorated.

Figure 3.7 compares cubic-smoothing spline estimates of G_t for three red spruce ring-width series using a 67$\%n$ criterion (solid line) and a fixed, 50% frequency-response cutoff of 60 years (dashed line). The 60-year cutoff is not the exact maximum SNR criterion, but it is much closer to it than the 67$\%n$ curves. The differences in the curve fits are readily apparent. The 67$\%n$ criterion does a poor job in tracking medium-frequency (10–30 years) fluctuations within each series that sometimes disagree with contemporaneous fluctuations in the other series. Examples of this lack of synchrony are found in the 1850s and 1950s of series (*b*). If signal is defined as information in the tree rings that is common to all trees in the ensemble, then the 67$\%n$ criterion does not eliminate noise as well as the 60-year cutoff. On the basis of noise reduction performance alone, the adequacy of the 67$\%n$ criterion will usually depend more on sufficient replication to average out endogenous-style growth fluctuations and on additional noise reduction techniques such as robust mean estimation and autoregressive modeling. All of these noise reduction techniques assume that common, exogenous non-climatic variance ($D2_t$) is not present in the ensemble. If $D2_t$ variance is known to be present either *a priori* or by strong inference, then more flexible filters may be necessary to remove that component.

The $\%n$ and maximum SNR criteria represent two reasonable and objective limits for selecting the frequency response of the digital filter and, therefore, the degree of flexibility of the resultant low-pass filter estimates of the age trend. If high-frequency information is of sole interest in a tree-ring chronology, then the latter criterion will probably be preferred if other noise reduction techniques, such as autoregressive modeling, are not used. However, if tree-ring series are to be used to study climatic and environmental change at virtually all resolvable wavelengths in a tree-ring chronology, then the $\%n$ criterion should be considered as a criterion of smoothness for the digital filter.

Although digital filtering, as described above, is attractive for estimating the non-climatic growth trend in ring-width series, a theoretical drawback of this method should be noted. The symmetry of the filter weights in equation (3.11) requires both past and future ring widths to estimate the central smoothed value for each year. This model for low-frequency changes in ring width is obviously incorrect because trees cannot possibly anticipate radial growth performance of future years in developing an expectation for ring width for the current year. The predisposition to grow at a certain potential rate must come only from the past. This means that one-sided or causal filters (Robinson and Treital, 1980) of the general form

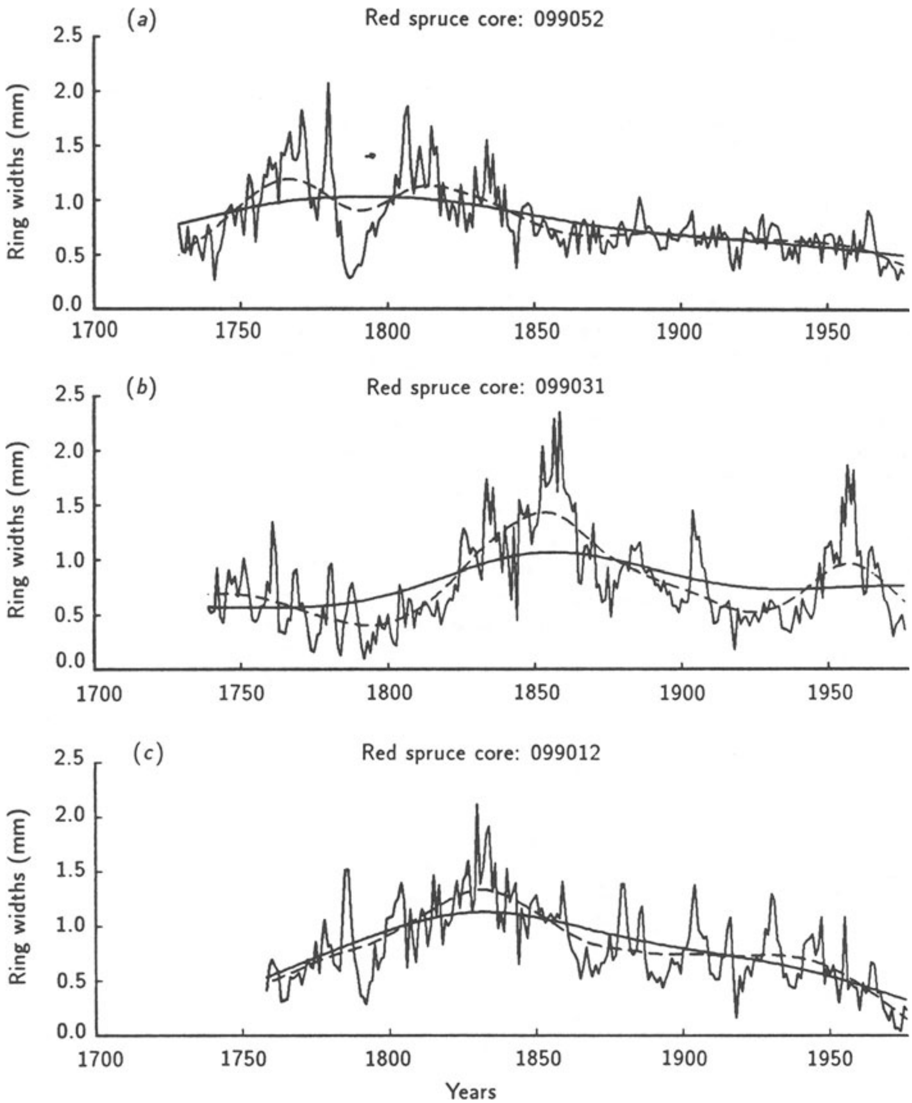

Figure 3.7. A comparison of two selections of smoothing ring-width data using the cubic-smoothing spline. The solid lines indicate spline values based on a 50% frequency-response cutoff of 67%*n*, where *n* is the series length in years. The dashed lines indicate splines with fixed 50% frequency-response cutoffs of 60 years. The more flexible curve fit is clearly superior on an individual series basis. However, with sufficient replication and the use of other noise reduction techniques, the %*n* criterion may be used if the need to preserve potentially resolvable long-term climatic variance is important.

$$Z_t = \sum_{i=1}^{-\infty} \Psi_i e_{t-i} + e_t \tag{3.16}$$

are more appropriate for modeling certain aspects of the persistence or predictability in ring width from year to year. Equation (3.16) is the general linear process, which serves as the foundation for autoregressive-moving average (ARMA) time series modeling (Box and Jenkins, 1970). The general linear process and its finite ARMA derivatives are causal filters because only current and past information is necessary for the evolution of the observed process. The use of ARMA models in tree-ring chronology development will be described more fully, later.

Exponential Smoothing

A quite different stochastic estimate of G_t is possible by using exponential smoothing (Barefoot *et al.*, 1974). The smoothing function chosen by Barefoot *et al.* (1974) consists of two components: an average, $\bar{R}_t$, and a lag correction for trend, $\tilde{R}_t$. This is expressed as

$$G_t = \alpha \bar{R}_t + (1 - \alpha)\, \alpha \tilde{R}_t \quad , \tag{3.17a}$$

where G_t is the smoothed estimate for year t,

$$\bar{R}_t = \alpha\,(R_t - 1) + (1 - \alpha)\,(\bar{R}_{t-1}) \quad , \tag{3.17b}$$

and

$$\tilde{R}_t = \alpha\,(R_t - R_{t-1}) + [(1 - \alpha)/\alpha]\,(\tilde{R}_{t-1}) \quad . \tag{3.17c}$$

The quantity α is a weighting factor that determines the degree of smoothing or how much past information on ring width enters into the current estimate. The influence of past ring widths decays exponentially as $(1 - \alpha)^n$, where n is the number of years prior to the current estimate. Barefoot *et al.* (1974) selected $\alpha = 0.2$ to smooth their ring widths, which allows the previous 10–15 years of data to influence the current estimate of G_t. In contrast to the symmetrical digital filter, exponential smoothing operates as a one-sided, causal filter (Robinson and Treital, 1980) that only relies on current and prior values in its estimation. As noted earlier, this is a desirable property in that the estimates of G_t evolve through time in the same way that trees grow. However, the selection of α may be series dependent and difficult to select. In this sense, it has the same problem as digital filtering.

Abraham and Ledolter (1983) review exponential smoothing techniques and suggest selecting an α that minimizes the one-step ahead prediction error variance. This criterion for α, if adequate, will produce a series of random residuals from the smoothing function. Except in rare cases where a serially random tree-ring series is of interest, an α based on minimizing the prediction mean-square error will remove too much low-frequency variance for climatic studies. The one-sided form of equation (3.17a) is conceptually appealing, but additional research is needed on developing objective guidelines for choosing the smoothing constant for tree-ring standardization.

Differencing

Another method of stochastic detrending is based on differencing (Box and Jenkins, 1970; Van Deusen, 1987). In this approach, a ring-width series is considered to be a random walk with deterministic drift. This process has the form

$$R_t = R_{t-1} + e_t + \delta \quad , \tag{3.18}$$

where R_t is the observed ring width, e_t is a serially uncorrelated random shock, and δ is the deterministic drift of the process that may also be thought of as a constant slope parameter. By taking first differences of R_t (usually after logarithmic transformation) as

$$\nabla R_t = R_t - R_{t-1} \quad , \tag{3.19}$$

the deterministic drift, which imparts linear trend to the R_t, is nothing more than the arithmetic mean of ∇R_t. However, the trend that is removed by differencing is actually stochastic, rather than deterministic. This is easily seen by noting that the conditional expectation of R_t given $R_{t-1}, R_{t-2,\ldots}$ is $E(R_t) = R_{t-1} + \delta$. Since R_{t-1} is subject to random shocks in the form of e_t, the deterministic trend component of $E(R_t)$ is also subjected to random shocks. Thus, the trend changes stochastically.

The advantage of differencing lies in its simplicity, causal structure, and total objectivity. There are no parameters to estimate, and the method is insensitive to ring widths more than one year apart. The latter property makes differencing especially attractive when old tree-ring collections are updated. The addition of new rings to each series will have no effect on the way that the detrended tree rings are produced. In this sense, differencing is locally robust as a detrending method when new data are added, which is not the case for deterministic least squares methods of detrending.

The disadvantage of differencing is the way in which the method acts as a high-pass filter. Virtually all low-frequency variance is attenuated and the high, year-to-year variance is emphasized. This property follows from the notion that the differenced series is nothing more than a numerical estimate of the 1stderivative of the process (Van Deusen, 1987). Thus, each differenced value is the relative rate of change in ring width from one year to the next. The severe attenuation of low-frequency variance in the first differences means that the resultant tree-ring chronology will not exhibit the year-to-year persistence commonly seen both in climate (Gilman *et al.*, 1963; Mitchell *et al.*, 1966) and in tree rings standardized by other detrending methods (Rose, 1983; Monserud, 1986). This deficiency can be ameliorated through the use of ARMA time series modeling techniques, which are described in the next section.

Differencing has only recently been applied to the problem of tree-ring detrending and standardization (Van Deusen, 1987; Guiot, 1987a). Because of this and its importance in autoregressive-integrated moving average (ARIMA) time series modeling (Box and Jenkins, 1970), it deserves additional research and testing.

3.3.4. Other methods of estimating growth trends

Other techniques can be used for estimating growth trends that do not clearly fit into the simple categories of techniques just described.

Graphic techniques. In the precomputer era of dendrochronology, growth trends were frequently estimated visually using flexible rulers (Schulman, 1956; Stokes and Smiley, 1968). Although this method has an inherently subjective aspect to it, it can be applied with a high degree of uniformity by experienced individuals when the growth trend is simple in form (i.e., negative exponential and linear). However, with the availability of digital computers and programs for estimating satisfactory growth trends, this method is rarely applied today.

Stand-level growth trends based on the biological age of trees. Another method of growth-trend estimation has been described by Erlandsson (1936), Mitchell (1967), and Komin (1987). It is based on collecting ring-width material from a large range of age classes of a given tree species. The ring-width measurements of each sample are aligned with those of the other samples according to the biological age of the rings, not the chronological age. For example, year five of a 200-year-old tree is aligned with year five of a 50-year-old tree by this method. Once the biological age alignment is done, the ring widths of all samples are averaged together to produce a tree-based, average biological growth trend. The averaging process greatly attenuates the yearly fluctuations in ring width due to environmental factors because of the chronological misalignment of the tree rings. Consequently, the underlying growth curve is emphasized. The degree to which the environmental effects are attenuated will depend on the sample size for each year, the distribution of tree ages in the collection, and the level of randomness through time of the environmental factors. The estimation of the mean growth curve also assumes that the structural form of the curve at any biological age is independent of the time period during which it is produced. There seems to be little room for endogenous and exogenous disturbance effects in this model.

Once the mean growth curve is estimated, a smooth mathematical function is fit to the curve and used to standardize the individual series from the site (Mitchell, 1967; Komin, 1987). Fritts (1976, page 280) points out that this method is flawed as a technique of tree-ring standardization. He notes:

> All individuals of a species rarely attain optimum growth at the same age, and individual trees differ in their growth rates because of differences in soil factors, competition, microclimate, and other factors governing the productivity of the site. Therefore, individual trees will deviate markedly and systematically from the mean growth curve.

These reasons are sufficient to reject the method as a standardizing tool for dendroclimatic studies. However, where the signal of interest is the variance within each series that deviates from the mean growth curve of the stand, this method is appropriate.

The corridor method. Shiyatov (1987) describes another approach to standardization called the corridor method. It is based on the construction of a maximum possible growth curve and a minimum possible growth curve for each ring-width series. These two curves form a growth corridor within which the range in ring-width variability fluctuates. *Figures 3.8(a)*, *3.8(c)*, and *3.8(e)* illustrate some common forms of the corridor, which is usually constrained to evolve smoothly through time. The maximum growth curve is based on a few ring widths that define the local maxima in the ring-width series. Typically, it displays the most pronounced growth trend and is usually constrained to have only one peak (Shiyatov and Mazepa, 1987). The minimum growth curve is based on a few ring widths that define the local minima in the series. It tends to have a less pronounced growth trend and may be constrained to repeat the bends of the maximum curve (Shiyatov and Mazepa, 1987). Where the ring widths get very small or when there is a high frequency of locally absent rings, the minimum curve approaches or equals zero [e.g., *Figures 3.8 (a)* and *3.8(c)*].

The tree-ring indices are calculated from the corridor estimates as

$$I_t = \frac{R_t - G1_t}{G2_t - G1_t} * 100\ (200) \quad , \qquad (3.20)$$

where I_t is the index, R_t is the ring width, $G1_t$ is the minimum growth-curve estimate, and $G2_t$ is the maximum growth-curve estimate, all for year t. The 100 (200) are scaling factors that transform the index into either percentages (* 100) of the corridor width or 2 × percentages (* 200).

A comparison of equation (3.20) with the more commonly used formula for computing indices, equation (3.1) reveals the difference in the techniques. The indices computed from equation (3.1) are (effectively) percentage departures from an expectation of growth (G_t). This is equivalent to using a time-dependent mean for standardizing ring widths. In contrast, indices computed from equation (3.20) are (effectively) percentages of the corridor width ($G2_t$ −

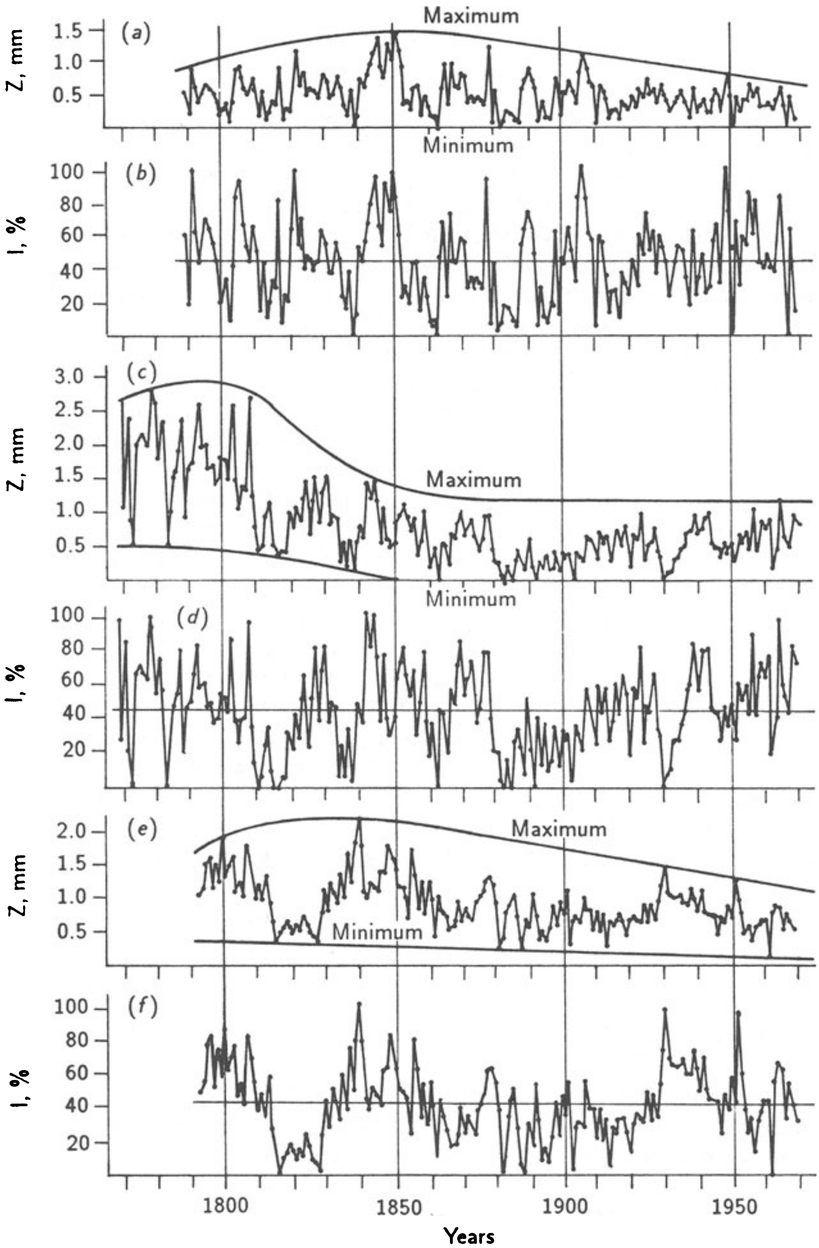

Figure 3.8. Examples of the *corridor* method of tree-ring standardization. The curves of maximum and minimum possible growth, which define the corridor, are shown for three Siberian larch (*Larix sibirica*) ring-width series [(*a*), (*c*), and (*e*)]. The standardized tree-ring indices derived from their respective corridors and equation (3.20) in the text are shown [(*b*), (*d*), and (*f*)].

Gl_t), which is equivalent to using a time-dependent range for standardizing ring widths.

Whether one selects the mean or the range may affect the limits that the resultant indices take. Indices computed by equation (3.1) are bound only by zero and have an unbound maximum. Although standardized tree-ring chronologies developed from equation (3.1) are usually normally distributed (Fritts, 1976), the lower bound means that the potential exists for a truncated or skewed probability distribution. In contrast, indices computed by equation (3.20) are bound by both the minimum and the maximum growth curves. If the minimum and maximum curves pass through an excessive number of ring widths, then the possibility exists for creating a probability distribution function of indices that is truncated at either or both ends of the distribution.

The corridor method also assumes that the time variations in the maxima and minima, which define the corridor, are principally biological in origin rather than climatic. This assumption follows from the way that the maximum (or minimum) values in the early and later segments of a ring-width series are constrained to be identical in indexed form, even though they may be quite different in absolute ring width. This constraint implicitly assumes that the growth-limiting factors owing to climate were identical for those years. Shiyatov (personal communication) notes that these issues are not likely to be problems in the mean-value function of corridor-indexed series because of the inherent between-tree variability of the ring widths used to define the corridors.

Figures 3.8(b), *3.8(c)*, and *3.8(f)* show the resultant indices computed from the estimated corridors in *Figure 3.8* and equation (3.20). It is clear that the technique produces indices having a stationary mean and homogeneous variance. Unlike indices computed from equation (3.1) that have an expected value of 1.0 or 100%, indices computed by the corridor method have an expected value of .5 or 50%. Shiyatov and Mazepa (1987) note that this difference can be eliminated simply by re-standardizing each corridor-indexed series using its long-term mean and equation (3.1).

The corridors shown in *Figure 3.8* were drawn by hand and human judgment using a flexible ruler. However, Shiyatov and Mazepa (1987) note that the technique can also be solved mathematically. They describe an estimation procedure that should make the corridor method more objective and far easier to implement as a standardization option.

Double-detrending. Holmes *et al.* (1986) describe a two-stage detrending method that they call *double-detrending.* A deterministic growth-trend model, such as the negative exponential curve, first estimates the observed trend in ring width. The tree-ring indices are computed from this curve and then detrended a second time using a cubic-smoothing spline. The second detrending is meant to remove any residual growth trend that is not modeled by the deterministic curve.

Holmes *et al.* (1986) justified double-detrending by illustrating that the negative exponential curve can fit the highly variable, steeply descending juvenile portion of the ring-width trend better than the less variable, flatter portion associated with maturity and old age. This is a consequence of least squares fitting, which can be dominated by the high-variance, juvenile portion of ring-width series and of the inadequacy of the negative exponential curve as a model for the observed growth trend. In such cases, the outer portion of a ring-width series may be systematically underfit or overfit for decades. Holmes *et al.* (1986) also showed that a stiff spline fit alone to the same series by the 67%n criterion was not sufficiently flexible to track the sharp curvature of the juvenile portion of the growth trend, even though it was quite adequate for the mature phase of the trend. Since each method of detrending was better for different portions of the growth trend, Holmes *et al.* (1986) reasoned that the sequential use of both techniques would correct the deficiencies of each method.

Cook (1985) examined the spectral properties of double-detrending and found that linear or negative exponential detrending followed by 67%n spline detrending worked well without removing too much low-frequency variance.

3.3.5. Concluding remarks on growth-trend estimation

The estimation and removal of growth trends from tree-ring series should be based, as much as possible, on the intended application of the tree-ring data. This means that there should be an *a priori* expectation of what the signal of interest is in the ring-width measurements. Given this expectation, the method of detrending should be chosen that will reduce the low-frequency noise not associated with that signal. It is difficult to accomplish this task within an objective framework because of the uncertainty distinguishing signal from noise in a given ensemble of ring-width data. Inevitably, some assumptions must be made that may have a great effect on the final standardized tree-ring chronology. It is imperative that these assumptions are carefully considered and justified in any application of standardized tree rings.

In general, stochastic methods are preferable to deterministic methods because of the freedom that the former possess in fitting the behavior of ring widths as they are observed, not as theory would have them behave. However, the consequence of this added flexibility are the problems of *ad hoc* model selection and overfitting, which are more likely to occur for stochastic models than for deterministic models. There also seems to be some utility in using a hybrid double-detrending approach, which can compensate for local lack-of-fit problems of single detrending methods.

3.4. Estimation of the Mean Chronology
E. Cook, S. Shiyatov, and V. Mazepa

3.4.1. Introduction

Once a collection of ring-width series has been detrended and indexed into a new ensemble of tree-ring indices, the estimation of the common signal, C_t, can proceed. As mentioned earlier, tree-ring indices can be treated as stationary, stochastic processes that allow them to be treated as a collective ensemble of realizations containing both a common signal in the form of C_t (and perhaps $D2_t$) and individual signals unique to the series ($D1_t$ and E_t).